Finding Local Extrema - The First and Second Derivative Tests

Math 130 - Essentials of Calculus

8 November 2019

Review - Increasing/Decreasing

Theorem

1. *If $f'(x) > 0$ on an interval, then $f(x)$ is increasing on that interval.*
2. *If $f'(x) < 0$ on an interval, then $f(x)$ is decreasing on that interval.*

Review - Increasing/Decreasing

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Example

Find the intervals on which the given function is increasing and decreasing

1. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

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Example

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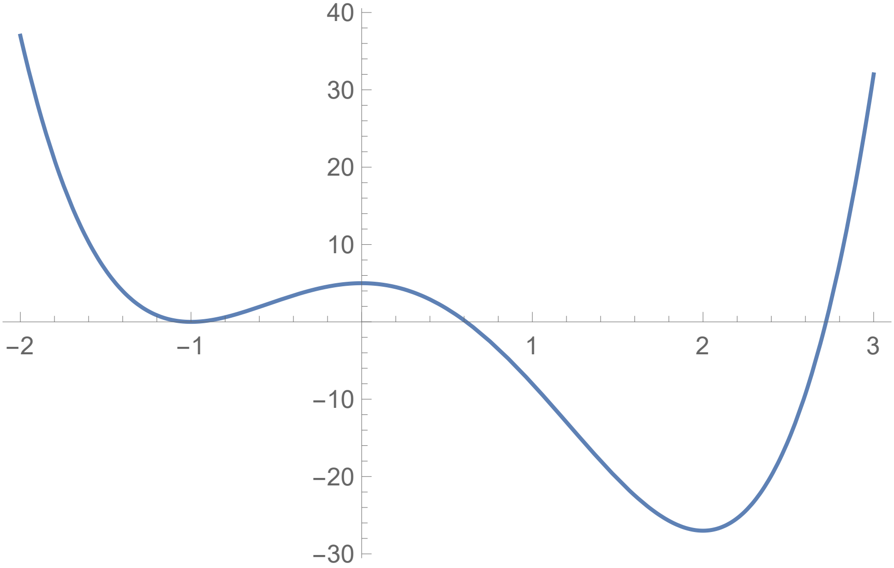
1. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
2. $f(x) = 2x^3 - 3x^2 - 12x$


40
30
20
10
−10
−20
−30
−2
−1
1
2
3

The First Derivative Test

Theorem (The First Derivative Test)

Suppose that c is a critical number of a continuous function f.

1. *If f' changes from positive to negative at c, then f has a local maximum at c.*
2. *If f' changes from negative to positive at c, then f has a local maximum at c.*
3. *If f' does not change sign at c (for example, if f' is positive on both sides of c or negative on both sides), then f has no local maximum or minimum at c.*

Example

Example

Find the local maximum and minimum values of $f(x) = 2x^3 - 3x^2 - 12x$.

Review - Concavity

Definition

A function $f(x)$

1. *is concave upward on a interval if f' is an increasing function on that interval.*

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Definition

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2. *is concave downward on a interval if f' is a decreasing function on that interval.*
3. *has an inflection point $x = c$ if f is continuous there and the concavity changes from upward to downward, or downward to upward.*

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Theorem

1. *If $f''(x) > 0$ on an interval, then $f(x)$ is concave upward on that interval.*
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Review - Concavity

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Concavity Example

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Find the intervals of concavity and inflection points for the given function.

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Concavity Example

Example

Find the intervals of concavity and inflection points for the given function.

1. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
2. $f(x) = 2x^3 - 3x^2 - 12x$
3. $y = x^4 - 4x^3$